p-Adic description of Higgs mechanism III: calculation of elementary particle masses

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6.10. 1994

Abstract

This paper belongs to the series devoted to the calculation of particle masses in the framework of p-adic conformal field theory limit of Topological GeometroDynamics. In paper II the general formulation of p-adic Higgs mechanism was given. In this paper the calculation of the fermionic and bosonic masses is carried out. The calculation of the masses necessitates the evaluation of degeneracies for states as a function of conformal weight in certain tensor product of Super Virasoro algebras. The masses are very sensitive to the degeneracy ratios: Planck mass results unless the ratio for the degeneracies for first excited states and massless states is an integer multiple of 2/3. For leptons, quarks and gauge bosons this miracle occurs. The main deviation from standard model is the prediction of light color excited leptons and quarks as well as colored exotic bosons. Higgsis absent from the spectrum as is also graviton: the latter is in accordance with the basic assumptions of p-adic field theory limit of TGD.

Contents

1 Introduction

This is the third paper in the series devoted to the p-adic description of Higgs mechanism in TGD [Pitkänen$_a$, Pitkänen$_b$] (for p-adic numbers see for instance [Brekke and Freund]). Concerning the general background reader is suggested to read the introduction of the first paper, where general formulation of p-adic conformal field theory limit was proposed and predictions are summarized. The general theory of Higgs mechanism was described in previous paper.

The calculation of masses of leptons, quarks and gauge bosons is carried out in this paper applying p-adic thermodynamics. The calculation of the masses boils down to the evaluation of degeneracies for states as a function of conformal weight in certain tensor product of Super Virasoro algebras. There are some delicacies associated with the norm of the states since inner product is defined as the massless limit of the inner product associated with the symmetry broken Super Virasoro representation. The thermal expectation values for fermionic masses are of order Planck mass for physical values of p-adic prime p unless the ratio for the degeneracies of $M^2 = 3/2$ Planck mass states and $M^2 = 0$ states is integer multiple of 2/3. It turns out that this miracle occurs for leptons and quarks as well as certain colored excitations of leptons and U type quarks. The physical consequences of the exotic light leptons and quarks are considered in the fifth paper of the series. Although there are hundreds of exotic bosons there are no massless noncolored exotic bosons so that no new long range forces are predicted. Higgs particle is absent from the spectrum as is also graviton: the latter in accordance with the basic assumptions about p-adic conformal field theory limit.

The predictions for lepton and gauge bosons masses agree surprisingly well with known experimental masses. Errors are below one per cent except for Z^0 boson for which mass is 10 per cent too large. The reason is too large value $sin^2(\theta_W) = 3/8$ for Weinberg angle: in the third paper it is shown that inclusion of Coulombic corrections and topological mixing effects of leptons leads to a correct prediction for gauge boson masses. One can safely conclude that the results of the calculation verify the essential correctness of both TGD and its p-adic conformal field theory limit at quantitative level. The detailed analysis and application of the results to derive information on of hadron masses is left to the fourth and fifth papers of the series.

Since the masses are very sensitive to the degeneracy ratios and since modulo arithmetics as well as the canonical correspondence between p-adic and real numbers is essentially involved, the details of the calculations are given in the form of appendices are listed so that interested reader can check them.

2 Calculation of elementary fermion and boson masses

In the sequel the calculations of elementary fermion and elementary boson masses are described at general level. The details of the calculations are left to appendix.

2.1 Modular contribution to the mass of elementary particle

The thermal independence of cm and modular degrees of freedom implies that mass squared for elementary particle is sum of cm and modular contributions:

$$M^2 \;=\; M^2(cm) + M^2(mod) \tag{1}$$

The general form of the modular contribution should be derivable from p-adic partition function for conformally invariant degrees of freedom associated with boundary components. The general form of vacuum state functionals as modular invariant functions of Teichmuller parameters was derived in [Pitkänen$_a$] and the square of the elementary particle vacuum functional can be identified as partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g > 0$ is given by

$$N(g) \;=\; 2^{g-1}(2^g + 1) \tag{2}$$

One has $N(1) = 3$ for muon and $N(2) = 10$ for τ.
e) Single theta function is analogous to partition function. This implies that

the modular contribution to mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in partition function.

f) The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g - g_1$ handles the partition function reduces to product of g_1 and $g - g_1$ partition functions. This implies that the contribution to mass squared is proportional to the genus of the surface. Altogether one has

$$
\begin{aligned}
M^2(mod, g) &= k(F)k(mod)2N(g)g\frac{M_0^2}{p} \\
k(F) &= 3/2 \\
k(mod) &= 1
\end{aligned}
\tag{3}
$$

Here $k(mod)$ is some integer valued constant (in order to avoid Planck mass) to be determined. $k(mod) = 1$ turns out to be the correct choice for this parameter. The value of the constant $k(F)$ in $M^2 = k(F)L_0 + ...$ is analogous to the string tension parameter and is determined uniquely from the requirement that actual tachyons are absent. Also the requirements that photons are massless, that charged leptons are light and that charged lepton mass ratios are predicted correctly, fix the value of this parameter to $k(B) = k(F) = 3/2$.

Summarizing, modular contribution to the mass of elementary particle (also boson!) belonging to $g + 1$:th generation reads as

$$
\begin{aligned}
M^2(mod) &= 0 \; for \; e, \nu_e, u, d \\
M^2(mod) &= 9\frac{M_0^2}{p(X))} \; for \; X = \mu, \nu_\tau, c, s \\
M^2(mod) &= 60\frac{M_0^2}{p(X)} \; for \; X = \tau, \nu_\tau, t, b
\end{aligned}
\tag{4}
$$

The formula can be tested also in hadronic context.

The higher order modular contributions to mass squared are completely negligible if the degeneracy of massless state is $D(0, mod, g) = 1$ in modular

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degrees of freedom as is in fact required by $k(mod) = 1$. The absence of vacuum degeneracy is natural assumption and guarantees lightness of boundary component for $g < 3$. This implies that second order contribution to mass squared is uniquely determined by cm contribution.

2.2 Calculation of cm contribution to fermion mass

Fermion masses are assumed be sums of cm and modular contributions $M^2 = M^2(cm) + M^2(mod)$. 'cm' refers to the cm of boundary component. The general form of modular 'contribution is determined completely by general arguments. The calculation of the cm contribution thermodynamically involves thermodynamics for Ramond type generalized spinors and in principle calculation is straightforward p-adic generalization of ordinary thermodynamics.

The possibility to describe Higgs mechanism thermally sets strong constraints on the theory.

a) M^4 and CP_2 degrees of freedom can be described using generalized H-spinor, which can be constructed as four-fold tensor products of $so(4)$ Super Virasoro representations. In $su(3)$' degrees of freedom $N - S$ type representation is assumed for leptons. This is possible since one can generalize the expression for super supper symmetry generators of Ramond type appropriately. The Ramond representation in $U(1)$ degrees of freedom implies doubling of degeneracies.

b) It turns out that $h = -5/2$ is the only possible value vacuum weight in case of neutrinos and $h = -3/2$ in case of charged leptons. The presence of tachyon is analogous to the presence of tachyonic Higgs particle (around symmetry nonbroken vacuum) in gauge theories. Tachyonic ground state implies vacuum degeneracy analogous to the degeneracy of symmetry broken vacua in gauge theories.

c) Tachyons can be eliminated either by direct constraint on spectrum or by defining the negative powers of p coming from tachyonic states as the limit $p^{-n} \equiv lim_{N \to \infty} p^{n(p-1)(1+p+p^2+...+p^N)} = 0$. Their elimination is not absolutely necessary since tachyonic states satisfying G-parity rule are actually particles with real mass for physically interesting primes p due to the special properties of p-adic square root (square roots of $-3/2$ and -3 are p-adically real). Furthermore, even states for which energy M is imaginary for the particle at

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rest allow states for which the components of four momentum are p-adically real.

d) Electroweak mass splitting is described by assuming that vacuum weight depends on isospin of the fermion via the formula

$$
\begin{aligned}
h(\nu) = h(U) &= -\frac{5}{2} \\
h(L^-) = h(D) &= -\frac{3}{2}
\end{aligned}
\tag{5}
$$

Since Kähler charge $Q_K = \pm 1$ increases vacuum weight by $Q_K^2/2 = 1/2$ one must use operators of weight $\Delta = -3/2 - I_3 + n$ to create $M^2 = n$ states and thermal expectations for charged leptons and neutrinos are different.

e) Only integer excitations of L_0 are allowed. For fermions half integer excitation transforms quark into lepton and vice versa and is excluded by triality rule of Quantum TGD (quarks and leptons correspond to triality one and zero representations of color group). The rule is just the G-parity rule of string models and in present case is necessary to exclude actual tachyons from spectrum.

f) No thermal mixing of different vectorial isospins takes place and thermalization occurs in Super Virasoro degrees of freedom only. Temperature parameter ($1/T$ is integer) is assumed to be $T = 1$ for fermions. The assumption $k(F) = 3/2$ is necessary for the elimination of tachyons by making their masses actually p-adically real and for prediction of correct leptonic mass ratios.

g) The thermal expectation for p-adic mass squared can be expressed in terms of degeneracies $D(i)$ of $M^2 = 0, 3/2, 3$ states in extremely good approximation as

$$
M^2(cm) = k(F)\frac{D(3/2)}{D(0)}p + k(F)\frac{(2D(3) - \frac{D(3/2)^2}{D(0)})}{D(0)}p^2
\tag{6}
$$

and the real counterpart of the mass squared is easily obtained from this expression using canonical correspondences between p-adic and real numbers. The task is to calculate the degeneracies.

h) If second order contribution to mass is written in the form $(X/64)p^2$ to mass squared correspond to small integer X then that ground state degeneracy must be $D = 64$ for fermions and $D = 16$ for bosons or more generally a power of 2 not larger than 64. It turns out that this condition is not realized.
i) The condensation levels for charged lepton families are assumed to be $k = 127, 113$ and $k = 107$ in accordance with the hypothesis about the importance of primes near prime powers of 2. A somewhat puzzling result is that the value of k for boundary component is same as for the interior: this is true not only for leptons but also for u, d, s quarks and hadrons ($k = 107$ for both). Since boundary contribution and interior contributions in principle correspond to two different condensation levels this makes sense only provided primary and secondary condensation levels correspond to nearly identical values of p. This mechanism was found to make possible the decrease of mass in condensation.

2.2.1 Calculation of cm contribution to quark mass

The calculation of cm contribution to quark mass differs from the leptonic case in some aspects.
a) The vacuum weights must be chosen so that essentially same operators create states of given mass squared for ν and U type quark and e and D type quark.
b) The contribution of Kähler charge $Q_K = 2/3$ of quark to ground state conformal weight is

$$\frac{Q_K^2}{2} = \frac{2}{9} \tag{7}$$

and gives Planck mass for the quark unless vacuum weight contains additional term of opposite sign cancelling the contribution. The first possibility is that quark confinement is related totally to color force rather than half odd integer charge so that vacuum weight contains compensating contribution

$$
\begin{aligned}
h(U) &= -2 - \frac{2}{9} \\
h(D) &= -1 - \frac{2}{9}
\end{aligned}
\tag{8}
$$

Quarks and leptons are essentially identical apart from effects caused by color force.

c) The second alternative to come in mind is based on the rather attractive idea that contribution of Kähler charge gives free quark Planck mass. In hadrons $Q_K(tot)$ is integer and light hadrons are obtained provided that

i) the contribution of Kähler charge is not additive but is given by $Q_K^2(tot)/2 = 1/2(0)$ for baryons (mesons) and

ii) the total vacuum weight of the baryon (meson) contains an anomalous contribution cancelling this contribution.

The assumption about additivity of vacuum weights for quarks doesn't allow this kind of scenario. Additivity in baryonic case implies $h_{anom}(q) = h_{anom}(\bar{q}) = -1/6$ and in leptonic case this would give $h_{anom} = -2/6$ and would give Planck mass for meson. The physical counter argument is that that quarks inside hadron are known to be massless and in p-adic thermodynamic scnenario this is realized if free quark is massless (the fraction of time spend in Planck mass states is given by p-adic Boltmann factor and of the order of $1/p!$)

d) Accepting the massless quark alternative the calculation of quark masses does not differ much form that for leptons and differences result from differences between N-S and Ramond type color Super Virasoro representations.

e) To calculate hadron masses one must use appropriate tensor product of Super Virasoro representations for quarks. Since quarks correspond to different boundary components they satisfy separately Super Virasoro gauge conditions. This simplifies enormously the calculations and to first order the quark masses are additive. One might wonder whether one should allow color excitations of quarks if they combine to form color singlet: the results of the third paper show that most hadrons would get Planck mass if this were the case. This means that free massless quark is the only possible alternative.

2.2.2 The identification of physical states and of inner product

Before considering the definition of the inner product it is useful to clarify the rules for identifying the physical states.

a) $so(4)/su(2)$ and $so(3,1)/su(2)$ coset representations decompose into tensor products of Super Virasoro representation and Kac Moody representation associated with $su(2)$ degrees of freedom. This implies that the operators creating degenerate states can be constructed as products of the genera-

tors L_i^n, G_i^n, where $i = 1, ..., 6$ labels the 2 $so(3, 1)$ representations , 2 $so(4)$ representations and $u(1)$ and $su(3)$ representations appearing in the tensor product.

b) The Virasoro conditions $L_n(tot)|phys\rangle = 0$ reduce to the four conditions

$$
\begin{aligned}
L_n|phys\rangle &= 0 \\
G_{-n+\epsilon}|phys\rangle &= 0 \ , n = 1, 2 \\
\epsilon(R) &= 0, \epsilon(N - S) = 1/2
\end{aligned}
\tag{9}
$$

in ordinary case. If the representation associated with particle tensor products of Ramond and N-S type representations then the conditions associated with Super generators G^k separate to independent conditions for N-S and Ramond representations.

c) The delicacies of the inner product force a formulation of gauge conditions different from the standard formulation. Physical states must also be orthogonal to the states of form $L^n O|vac\rangle$ and Virasoro conditions can be replaced with the ortogonality requirement.

The definition of a physically sensible inner product $so(3, 1)$ and $so(4)$ degrees of freedom requires careful considerations. In $so(3, 1)$, $so(4)$ and $su(3)$ these degrees of freedom as well in color degrees of freedom the representations in question have $(c, h) = (0, 0)$. Formally this implies in $so(3, 1)$ and $so(4)$ degrees of freedom that the states created by Virasoro algebra generators $L_n(i)$ from vacuum state have vanishing norm. Only the ground state would correspond to nongauge degree of freedom! This is certainly not the physical situation and the problem derives from the fact that naive inner product is not correct. In $su(3)$ degrees of freedom zero norm for the states created by Virasoro generators seems to be make sense.

In p-adic case there is an elegant manner to modify the inner product in $so(3, 1)$ and $so(4)$ degrees of freedom. The point is that in p-adic case it is sensible to consider the limit $m \to 2$ by assuming m to be integer assuming that the values of P and Q are just those associated with the massless representations. One simply takes m to be integer of form $m = 2 + O(p^k)$ and allows k to approach infinity. In this limit the central charge behaves as

$$c = \frac{9\Delta m}{8}$$
$$\Delta m = O(p^k), \ k \to \infty \qquad (10)$$

The vacuum weight h behaves as

$$h(Ramond) = \frac{3\Delta m}{64}$$
$$h(N - S, 1, 1) = 0$$
$$h(N - S, 1, 3) = \frac{\Delta m}{4} \qquad (11)$$

Inner products in Virasoro algebra at $m = 2$ limit can be defined using the limiting expressions for c and h in the expressions for inner products and dividing by the small parameter Δm: just scaling is in question. Alternatively, one can consider the inner products for states, whose norm is taken to be equal to one. The inner product is unitary for each value of Δm in the limit and therefore unitarity holds true in the limit $\Delta m = 0$, too.

The inner products involve typically commutators/anticommutators of various Super Virasoro generators and it is useful to list the action of the commutators on the vacuum

$$Comm(L^m, L^{-m}) = 2mh + \frac{c}{12}m(m^2 - 1)$$
$$Anti(G^m, G^{-m}) = 2h + \frac{c}{3}(m^2 - \frac{1}{4})$$
$$(12)$$

The detailed study of these commutators shows that
a) G^0 creates zero norm state for Ramond representation in the limit $\Delta m \to 0$.
b) G^k and L^1 create zero norm state in case of $N - S$ $(1, 1)$ representation (vanishing spin/isospin).
These operators clearly act as super gauge symmetries. The natural physical interpretation is as remnant of the gauge symmetries defined by the entire Super Virasoro algebra.

The crucial property of the inner product is that for Ramond representation state space decomposes into sectors according to the number N of excited *so...* sectors and superposition for states belonging to different sectors does not make sense since without normalization inner products are of different order in Δm in different sectors and superposition with norm taken to be equal to one would imply that some coefficients in the superposition have infinite value. Therefore one must pose selection rule forbidding the superposition of states belonging to different sectors. Similar phenomenon occurs in bosonic sector.

The correct manner to treat the gauge conditions is to calculate their consequences for $c, h \neq 0$ and to take the limit $\Delta m \to 0$ only after that. Furthermore, one must consider matrix elements between physical states and states created by $\Delta = 2$ operators proportional to Super Virasoro generators. The delicacies associated with the definition of the inner product turn out to be important in the treatment of fermionic gauge conditions. It turns out that one must weaken the standard form of gauge conditions in $N > 1$ super selection sectors of state space.

Since the generators of Super Virasoro algebra in question are generated as commutators and of the generators L^2, L^1, G^1 one can restrict the consideration to these gauge conditions. This automatically eliminates redundancy associated with gauge conditions and simplifies practical calculations considerably.

2.2.3 Super selection rule

The emergence of super selection rule can be understood by studying the general form of the neutrino state in massless case. The general solution to the gauge conditions can be written as

$$
\begin{aligned}
O &= O_1 + O_2 + O_3 + O_5 + O_c \\
O_1 &= \sum_{i=1}^{4} (a(i)L_i^2 + b(i)(L_i^1)^2 + c(i)G_i^2 + d(i)L_i^1 G_i^1) \\
O_2 &= \sum_{i} (e(i)L_i^1 L_5^1 + f(i)L_i^1 G_5^1 + g(i)G_i^1 L_5^1 + h(i)G_i^1 G_5^1)
\end{aligned}
$$

$$\begin{aligned}
O_3 &= \sum_{i \neq j} (a_{ij} L_i^1 L_j^1 + b_{ij} G_i^1 G_j^1 + c_{ij} L_i^1 G_j^1) \\
O_5 &= e L_5^2 + f(L_5^1)^2 + g G_5^2 + h L_5^1 G_5^1 \\
O_c &= a_c F^{3/2a} F^{1/2a}
\end{aligned} \tag{13}$$

The requirement that the norm of each component in the state is nonvanishing and finite fixes the dependence of the various coefficients on the parameter Δm. One has the following proportionalities:

$$\begin{aligned}
a(i), b(i), c(i), d(i) &\propto \frac{1}{\sqrt{\Delta m}} \\
e(i), f(i), g(i), h(i) &\propto \frac{1}{\sqrt{\Delta m}} \\
a_{ij}, b_{ij}, c_{ij} &\propto \frac{1}{\Delta m} \\
a_5, b_5 &\propto 1
\end{aligned} \tag{14}$$

The coefficients become singular at the limit $\Delta m \to 0$ and that the dependence on Δm is different for the various states appearing in the decomposition ($a(i)...., h(i) \propto 1/\sqrt{\Delta m}$, $a_{ij}, b_{ij}, c_{ij} \propto 1/\Delta m$. The states with different powers of Δm are however automatically orthogonal and if one assumes the superselection rule forbiding linear superpositions of states belonging to different sectors there is no need to use diverging coefficients if matrix elements are defined projectively by dividing the inner product of two states with the norms of the states.

2.2.4 Results for lepton and quark masses

The results for the leptonic degeneracies are summarized in the following table

n	0	1	2	X_1	X_2
D(L)	12	40	80	5	$\frac{2}{3}$
D(ν)	40	80	10	3	$\frac{1}{2}$
D(D)	12	40	80	5	$\frac{2}{3}$
D(U)	40	80	8	3	$\frac{1}{2}$

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Table 2.1. The degeneracies $D(i)$, $i = 01, 2$ for charged leptons and neutrinos and quarks. Also are listed the coefficients $X_1 = k(F)D(3/2)/D(0)$ and $X_2 \equiv ((3(D(3) - D(3/2)^2/2D(0)) \ mod \ D(0))p^2)/D(0)$ of first and second order contributions to mass squared.

For the choice

$$k(F) \ = \ k(B) = \frac{3}{2} \tag{15}$$

corresponding to mass formula $p^2 = (3/2)L^0 + ...$ the coefficient $X_1 = \frac{k(F)D(3/2)}{D(0)}$ of first order contribution to mass squared is integer and one has $X_1 = 5$ for charged leptons and $X_1 = 3$ for neutrinos. It is rather remarkable that the condition $k(B) = 3/2 = k(F)$, which guarantees massless photon and predicts charged lepton masses with relative error smaller than one per cent also guarantees lightness of charged leptons!

For charged lepton the coefficient of second order contribution to mass squared is $X_2 = 2/3$. For Mersenne primes the real counterpart of $2p^2/3$ is $\frac{2}{3p}$ and same result is obtained under rather general assumptions about p associated with primary condensation level. For the neutrino one has $X_2 = -1/2$. The real counterpart of $-p^2/2$ depends on the value of $p(\nu)$: for Mersenne prime M_{127} the real counterpart is in good approximation $\frac{1}{2p}$. Similar result holds true under rather general conditions on $p(\nu)$.

The coefficients of first order contributions to mass squared are identical for leptons and quarks. The difference in second order contribution of U and ν results from the difference between Ramond and N-S type color Super Virasoro algebras. The over all mass scale for quark masses is determined by the condensation level, which must correspond to prime $k = 107$ for u, d, s and $k = 103$ for c and b as will be found in the third part of the paper.

Summarizing, under rather general conditions on p associated with the primary condensation level the expression for the cm contribution to lepton mass and its real counterpart reads as

$$M^2(cm, L) \ = \ (5p + \frac{2}{3}p^2)M_0^2$$

$$M^2(cm, L)_R = (5 + \frac{2}{3})\frac{M_0^2}{p(L)}$$

$$M_R^2(cm, \nu) = (3p + \frac{27}{20}p^2)M_0^2$$

$$M_R^2(cm, \nu)_R = (3 + \frac{7}{10})\frac{M_0^2}{p(\nu_e)} \qquad (16)$$

For electron no modular contribution is present so that a prediction for electron gauge boson mass ratio follows:

$$\frac{M_W}{m_e} = \sqrt{\frac{M_{127}}{M_{89}}}\sqrt{\frac{1}{2(5 + \frac{2}{3})}} \qquad (17)$$

W mass ($m_W \simeq 80.8\ GeV$ experimentally) is predicted to be too small by 0.7 per cent. Z^0 mass is predicted too large by 10 per cent and th error derives the too large value of Weinberg angle ($sin^2(\theta_W) = 3/8$). Topological condensation is expected to renormalize Weinberg angle by reducing Z^0 mass.

There are also some exotic light fermions. The calculations of the appendix show that all leptons allow color decuplets (10 and $\bar{10}$) and neutrinos also twice degenerate 27-plet as massless state. U type quarks allow also massless color decuplets. The masses of color excitations given in the table below.

fermion	$M_R/m_e\sqrt{\frac{M_{127}}{p}}$
L^{10}	$\sqrt{\frac{9}{5+\frac{2}{3}}}$
ν_L^{10}, ν_L^{10}	1
ν_L^{27}	$\sqrt{\frac{9}{5+\frac{2}{3}}}$
$U^{10}, U^{\bar{10}}$	$\sqrt{\frac{9}{5+\frac{2}{3}}}$

Table 2.2. The masses of color excited leptons and quarks.

The existence of colored electron implies new branch of physics unless the primary condensation level for some reason corresponds to small p rather than M_{127}. Recall that the TGD inspired explanation for anomalies e^+e^-

pairs observed in heavy ion collision [Pitkänen₁, Pitkänen and Mähonen] is in terms of leptopions, which are color bound states of color excited leptons.

2.3   Calculation of bosonic masses

Elementary boson mass squared is assumed to be given by boundary contribution, which can be expressed as sum of cm and modular contributions: $M^2 = M^2(cm) + M^2(mod)$. The fact that higher boson families are not observed does not necessary implies Planck mass for higher boson families. The construction of elementary particle vacuum functionals [Pitkänen$_a$] demonstrated that the amplitudes for transitions such as the decay of muon to electron by emission of $g = 1$ boson vanish for vacuum functionals. If $g > 0$ families are light their mass is at least of the order of intermediate gauge boson mass if one takes seriously the observation that the absence of tachyons for bosonic Dirac operator leaves only M_n,$n = 89, 61, 17$.

The construction of the cm contribution $M^2(cm)$ to bosonic mass relies on the following physical picture.
a) Bosons are described using generalized spinors and obey generalized Dirac equation leading to mass shell condition $p^2 = k(B)L^0+$. As already found the absence of actual tachyons is guaranteed if one has $k(B) = k(F) = 3/2$. Same result follows also from the requirement that photon is massless.
b) Operators with half odd integer conformal weight transform leptons to quarks and vice versa. Leptoquarks, that is bosons transforming leptons to baryons and vice versa, are not possible at classical level and G-parity rule motivated by tachyon elimination excludes these states. The rule also guarantees that partition function contains only integer powers of p (rather than half integer powers).
c) The temperature parameter $T = 1/n$, $n = 1, 2, 3...$ appearing in the partition function is a free parameter at this stage, which could even depend on particle. It turns out that $T(ew) = 1/2$ is necessary to understand the masses of electroweak gauge bosons. $T = 1$ provides a manner to eliminate exotic massless states from spectrum. As a consequence, the contribution to the thermal masses from $\Delta = 1$ level is of order $O(1/p^2)$ for gauge bosons. The condition $D(3/2) \ mod \ D(0) = 0$ guarantees that boson is essentially massless.

Consider next the technical aspects of the state construction.

a) Boson state must contain information about its couplings to fermions. It is useful to separate Super Virasoro and Kac Moody degrees of freedom, where charge matrices act. Bosonic charge matrices are linear combinations of CP_2 sigma matrices and Kähler charge times unit matrix. The Kac Moody counterparts for vectorial and axial isospins and Kähler charge are the matrices $F_3^{1/2,3}$, $F_4^{1/2,3}$ and $F_5^{1/2}$. The counterparts of W boson charge matrices are linear combinations of the matrices $F_3^{1/2,i} F_4^{1/2,j}$, $i,j = 1,2$. For gluons the charge matrices are the operators $F^{A1/2}$, $A = 1,..,8$. Bosons are therefore generated from ground states of form $|phys\rangle = EQ_B^k|vac\rangle$, $E = \epsilon_k \gamma_{1/2}^k$. For spin one bosons the state is also proportional to the operator E of conformal weight $1/2$ defined by the polarization vector of gauge boson. The state satisfies automatically gauge conditions. Massless states are obtained by applying operators of conformal weight $\Delta = 2 - k$ to the ground state. The generalized Dirac equation implies the conditions $p^2 = 0$ and $p \cdot \epsilon = 0$ for the massless states.

b) It is not clear whether one should allow all isospins for the operators $F_k^{1/2,i}$ (isospin index has not been written in the formulas of appendix). The fact that intermediate gauge bosons correspond to subset of all possible index combinations suggests that there is some hitherto unidentified condition excluding some isospins. This kind of rule might be the analogy of Dirac equation in $so(3,1)$ degrees of freedom and would mean the introduction of $so(4)$ momentum and polarization. $so(4)$ polarization could have something to do with the direction defined by the vacuum expectation value of Higgs field in standard model. Unfortunately, the situation is unclear at this stage.

c) For intermediate gauge bosons also longitudinal polarization operator $P = p_k \gamma_k^{1/2}$ is present in massless sector since otherwise longitudinal polarization would correspond to Planck mass excitation. It is necessary to define the norm of longitudinally polarized massless state by dividing with $\sqrt{p^2}$ and taking the limit $p^2 \to 0$. This indeed makes sense if one defines inner product in $so(4)$ degrees of freedom as the p-adic limit $m = 2 + \Delta m \to 2$ so that $p^2 \propto h \propto \Delta m$ holds true. In the actual physical situation a small value of Δm might well be generated by secondary topological condensation so that limiting procedure could be regarded as a useful mathematical idealization. The counterpart of this in ordinary description of massivation is the transformation of the gradient of Higgs field to the longitudinal polarization of gauge boson. Since P is not used in state construction for transversally polarized

states there is complete symmetry between transversal and longitudinal polarizations and the calculations performed for transversal polarization apply as such for longitudinal polarization. One can choose the coordinates so that polarization vector corresponds to $i = 2$ tensor factor and $P = p_k \gamma^k_{1/2}$ corresponds to $i = 1$.

d) The operators creating massive excitations are just the Super Virasoro generators associated with different tensor factors in $so...$ degrees of freedom. In color degrees of freedom the multiplication with Super Virasoro generators produces zero norm states and one must use commutator action instead. An alternative possibility is that no excitations are allowed in color degrees of freedom. In color degrees of freedom there is possibility of forming nonlinear combinations of Kac Moody generators, which would imply nonlinear terms in couplings to fermions. It turns out that massless gluon is obtained if only linear combinations are allowed.

e) In fermionic case it was necessary to modify the inner product for Super Virasoro, which led to the decomposition of the state space to super selection sectors labeled by $N = 0, 1, 2, ..$ Also in bosonic case one can consider the possibility of including the states created by L^n $n \geq 2$ and $G^k, k \geq 3/2$ from N-S singlet vacuums in $so...$ degrees of freedom. For ordinary inner product these states possess vanishing norm but one could modify the norm in the same manner as in fermionic case and obtain additional super selection sectors to the state space. The construction of boson states serves as test for this alternative. It has not been possible to identify any working scenario allowig the inclusion of $N \geq 0$ sectors of state space. It should be noticed that for $u(1)$ sector the states created by polynomials of $L^k, k \geq 0$ and $G^k, k \geq 3/2$ possess nonvanishing norm as is clear from the commutator algebra ($G_5^{3/2}$ creates state with nonvanishing norm!) and must be taken into account in state construction.

The results are listed in the tables and will be discussed in detail in subsequent paper. The masses of gauge bosons are in quantitative accordance with expectations assuming $T(ew) = 1/2$ whereas $T = 1$ must be assumed for exotic bosons. The remarkable result is the absence of massless exotic bosons.

boson	M_{obs}/MeV	M_{pred}/MeV	error/%
γ	0	0	0
gluon	0	0	0
W	80200	79582	-0.8
Z	91151	100664	10.0

Table 2.3. Masses of nonexotic gauge bosons.

spin	charge operator	$M^2(T=1)$	$M^2(T=1/2)$
0	$I^3_{L/R}$	Planck mass	$\frac{1}{2p}$
0	$I^{\pm}$	Planck mass	$\frac{1}{2p}$
0	$I^3_{L/R}Q_K$	$\frac{3}{p}$	0
0	$I^{\pm}Q_K$	Planck mass	$\frac{1}{2p}$
1	1	Planck mass	$\frac{1}{2p}$

Table 2.4. Masses and couplings of noncolored light exotic bosons for $T = 1$ and $T = 1/2$. Charge operator tells how the boson in question couples to matter. For $T = 1/2$ the states with charge operator $I^3_{L/R}Q_K$ are essentially massless for large values of p and some additional light states become possible.

spin	charge operator	D	$M^2(T=1)$	$M^2(T=1/2)$
0	$I^{\pm}Q_K$	8	$\frac{2}{p}$	0
1	$I^{\pm}$	8	Planck mass	$\frac{1}{2p}$
1	$I^3_{L/R}Q_K$	8	Planck mass	$\frac{1}{2p}$
1	$I^{\pm}Q_K$	8	Planck mass	$(\frac{3p}{10})_R\frac{1}{p}$
0	$I^{\pm}Q_K$	$10,\bar{10}$	$\frac{3}{p}$	0
0	1	$10,\bar{10}$	0	0
1	$I^{\pm}$	$10,\bar{10}$	$\frac{3}{p}$	0
1	$I^3_{R/L}Q_K$	$10,\bar{10}$	$\frac{3}{p}$	0
0	$I^{\pm}$	27	Planck mass	$\frac{1}{2p}$
0	$I^3_{R/L}Q_K$	27	Planck mass	$\frac{1}{2p}$
1	$I^3_{R/L}$	27	Planck mass	$\frac{1}{2p}$
1	Q_K	27	Planck mass	$\frac{1}{2p}$
0	$I^3_{R/L}$	27	0	0
0	Q_K	27	0	0
1	1	27	0	0

Table 2.5.   Masses and couplings of colored exotic bosons for $T = 1$ and $T = 1/2$. The last two massless bosons are doubly degenerate due to occurrence of two 27-plets with conformal weight $n = 2$. $T = 1/2$ is physically possible alternative since no long range forces are implied.

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3   Appendix A:Calculation of degeneracies for neutrinos and U type quarks

In the following the thermal expectation for the cm contribution to mass squared of neutrino and U quark is derived. The differences between neutrino and U quark derive from the differences between Ramond (quarks) and N-S type (leptons) representations of Super Virasoro in color sector. The first difference comes in order $O(p^2)$. The results of calculation can be used in the considerably simpler calculation of $M^2(cm)$ for charged leptons and D quarks.

Supersymmetry requirement leads to some delicate considerations for L^2 gauge conditions necessitated by the limiting procedure appearing in the definition of the inner product. States have well defined number of $so(4)$ type supergenerators G_i^k , which is odd or even. Supersymmetry means that the solutions with odd number of super generators are obtained by the applicatication of G^0 from the solutions with even G-parity. This implies that in certain cases the gauge conditions for L^2 implies two conditions instead of one condition as one might expect. The point is that L_i^0 terms in state L^2X give scalar term proportional to $h(\to 0)$ and the coefficient of this term must vanish. Also terms proportional to G_i^0 appear in the state L^2X and give zero norm state. The application of G^0 to the state L^2X however transforms these terms to terms proportional to L_i^0 and implies doubling of L^2 gauge condition. The resulting gauge condition is just that for G^2 and would follow for ordinary inner product as a consequence of L^1 and G^1 gauge conditions.

The following tables summarize the results for the degeneracies for various mass squared values in various super selection sectors.

M^2	N=0	N=1	N=2	N=3	N=4	D
0	0	20	20	0	0	40
$\frac{3}{2}$	0	44	32	4	0	80
3	2	8	0	0	0	10

Table 3.1. The degeneracies of neutrino states with $M2 = 0, 3/2, 3$ in super selection sectors with $N = 0, 1, 2, 3, 4$. Last column gives total degeneracies.

M^2	N=0	N=1	N=2	N=3	N=4	D
0	0	20	20	0	0	40
$\frac{3}{2}$	0	44	32	4	0	80
3	0	8	0	0	0	8

Table 3.2. The degeneracies of U quark states with $M^2 = 0, 3/2, 3$ in super selection sectors with $N = 0, 1, 2, 3, 4$. Last column gives total degeneracies.

3.1 Degeneracy of $M^2 = 0$ states of U quark and neutrino

Super selection rule implies that gauge conditions can be applied separately in each sector.

1. $N = 0$ sector.

For $N = 0$ states the gauge conditions correspond to 4 operators O_5^2 (4) and the operator $O_c^2 = \sum_a F^{a3/2} F^{a1/2}$ acting in color degrees of freedom: 5 altogether. This operator is present also in quark sector. Gauge conditions for L^1 and G^1 correspond to operators $O_5^1(2)$ and gauge condition for L^2 corresponds to a multiple of unit operator (1). The number of gauge conditions is $2 + 2 + 1 = 5$ so that no solutions to gauge conditions are obtained: $D(0, 0) = 0$.

2. $N = 1$ sector

The 32 states in $N = 1$ sector are created
i) by 16 single particles operators O_i^2 given by L_i^2, $(L_i^1)^2$, G_i^2, $L_i^1 G_i^1$.
ii) by 16 2-particle operators $O_{i5}^{1,1}$ with O_{i5} given by $L_i^1 L_5^1$, $L_i^1 G_5^1$, $G_i^1 L_5^1$, $G_i^1 G_5^1$, $i \leq 4$ coupling $so(3,1) \times so(4)$ and $u(1)$ degrees of freedom to each other.

Gauge conditions for L^1 and G^1 correspond to operators $O_i^1, i = 1, ..., 5$, $(5 \cdot 2 = 10)$ and gauge conditions for L^2 correspond to unit matrix. The orthogonality with respect to states created by the operators L^2 gives actually two conditions since G^0 acts as super symmetry and states can be classified in two types with general forms

$$O = \sum_i (a_i L_i^2 + b_i (L_i^2)^2 + c_i L_i^1 L_5^1 + d_i L_i^1 G_5^1)$$

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$$O = \sum_i (a_i G_i^2 + b_i L_i^1 G_i^1 + c_i G_i^1 L_5^1 + d_i G_i^1 G_5^1)$$

(18)

and the condition comes from the coefficient of L^0 term in commutator. The number of conditions is therefore 22. This gives $32 - 22 = 10$ solutions to gauge conditions and taking into account the doublefold degeneracy associated with $U(1)$ degrees of freedom one has $D(0,1) = 20$.

3. $N = 2$ sector.

There are 24 two-particle operators $O_{ij}^{1;1}$, $i \neq j$ where $O_{ij}^{1;1}$ is given by $L_i^1 L_j^1$, $L_i^1 G_j^1$, $G_i^1 L_j^1$, $G_i^1 G_j^1$, $i \neq j \leq 4$. The application of gauge conditions for L^1 and G^1 gives

$$\sum_j a_{ij} = \sum_j b_{ij} = \sum_j c_{ij} = \sum_j c_{ji} = 0$$

(19)

and the number of operators is reduced to 10. Naiive counting of gauge conditions would give $8+8 = 16$ gauge conditions corresponding to operators O_i^1 associated with L^1 and G^1 gauge conditions. The reason for redundancy is that for the antisymmetric part of the coefficient matrix the gauge conditions are redundant. To sum up, one has $D(0,2) = 20$. and $D(0) = 40$. There are no differences between U quark and neutrino.

3.2 Degeneracy of $M^2 = 3/2$ states for neutrino and U type quark

$M^2 = 3/2$ states are created by the operators of conformal weight $\Delta = 3$ from a Ramond ground state with definite quantum numbers. $M^2 = 3/2$ operators can be labeled by the number N defined as the number of *so...* type Super Virasoro generators appearing in them and linear super position for states with different N is forbidden by super selection rule. N can have the values $N = 0, 1, 2, 3$ for $M^2 = 1$ states. It is useful to introduce some shorthand notations. O_i^n refers to $\Delta = n$ operators in sector $i = 1, ..., 5$. O_i^1 refers to L_i^1 and G_i^1. O_i^2 refers to the 4 operators $L_i^2, G_i^2, (L_i^1)^2, L_i^1 G_i^1$. O_i^3 refers to the 8 operators $L_i^3, G_i^3, L_i^2 L_i^1, (L_i^1)^3, G_i^2 G_i^1, L_i^2 G_i^1, (L_i^1)^2 G_i^1, L_i^1 G_i^2$.

1. $N = 0$ sector.

$N = 0$ operators correspond to operators acting in $u(1) \times su(3)$ degrees of freedom only. There are following operators.

a) The 8 operators O_5^3 acting in $u(1)$ degrees of freedom.

b) Color singlet operators $O_c^3(2)$ satisfying gauge conditions. From the table 8.3 of appendix F one finds that there are no color singlet operators satisfying gauge conditions.

c) The operators $O_5^1 F^{a3/2} F^{a1/2}(2)$.

Total number of operators is $8 + 2 = 10$.

The total number of gauge conditions is $4 + 4 + 2 = 10$ and is same larger than the number of operators so that one has $D(1, 0) = 0$. Same calculation applies to U quark since also in this case there is one color singlet operator O_c^2 satisfying all gauge conditions except the gauge conditions associated with L^2 (see table 8.3.

2. $N = 1$ sector.

In $N = 1$ sector there are following operators.

a) The 32 operators O_i^3, $i = 1, ..., 4$ acting in $so...$ degrees of freedom.

b) The 32 operators $O_i^2 O_5^1$ and the 32 operators of of form $O_i^1 O_5^2$.

c) The 8 operators $O_i^1 F^{a3/2} F^{a1/2}$, $i = 1, ..., 4$ acting in $so... \times su(3)$ degrees of freedom. These operators satisfy all gauge conditions associated with G^k, $k = 1/2, .., 5/2$. The only nontrivial gauge conditions are associated with L^1 and L^2.

The total number of $N = 1$ operators is 104.

a) Gauge conditions for L^1 and G^1 correspond to i) the 32 operators O^2 in $N = 1$ sector. There is a rather delicate reduction of gauge conditions associated with G^1: these gauge conditions are equivalent with the requirement that physical states are orthogonal to states of form $G^1 O^2$ and for the states $O^2 = G_i^1 G_5^1$, $i = 1, .., 4$ the ortogonality condition is identically satisfied since the projection of O^2 to $N = 1$ sector vanishes automatically by anticommutativity of G_i^1. This implies the reduction of gauge conditions by 4 to 28.

ii) the 5 operators O^2 in $N = 0$ sector. These operators were already listed, when evaluating the degeneracy of massless states.

Altogether thre are $32 + 28 + 10 = 70$ conditions.

b) Gauge conditions for L^2 correspond to the 8 operators O_i^1 and 2 operators O_5^1. The requirement of supersymmetry doubles the gauge conditions associated with O_5^1 so that the number of conditions is actually $8 + 4 = 12$.

The total number of gauge conditions is $32 + 28 + 10 + 12 = 82$ and the number of physical operators is $104 - 82 = 22$. The contribution to degeneracy is $D(1, 1) = 44$.

3. $N = 2$ sector.

There are following $N = 2$ operators.
a) The operators $O_i^2 O_j^1$, $i \neq j = 1, ..., 4$, whose total number is 96.
b) The operators $O_i^1 O_j^1 O_5^1$, $i \neq j = 1, ..., 4$, whose total number is 48.
The total number of operators is $96 + 48 = 144$.

a) Gauge conditions for L^1 and G^1 correspond to
i) 24 operators $O_i^1 O_j^1$ in $N = 2$ sector.
ii) 16 operators $O_i^1 O_5^1$ and 16 operators O_i^2 in $N = 1$ sector: altogether 32 operators.
The total number of conditions is $2 \cdot 56 = 112$.
b) Gauge conditions for L^2 correspond to 8 $N = 1$ operators O_i^1. The number of conditions get doubled by super symmetry.
This means that the number of conditions becomes $112 + 16 = 128$ and the number of states becomes $144 - 128 = 16$ and the contribution to the degeneracy is $D(1, 2) = 32$.

4. $N = 3$ sector.

32 operators $O_i^1 O_j^1 O_k^1$, $i \neq j \neq k = 1, .., 4$ contribute to the degeneracy in $N = 3$ sector. For the states $a_{ijk} G_i^1 G_k^1 G_k^1$ and $b_{ijk} G_i^1 G^j G_k^1$ gauge conditions allow one solution for each and the the contribution to degeneray is $D(1, 3) = 4$. The total degeneracy of $M^2 = 3/2$ states is $D(3/2) = 80$. Same result is obtained for U quark.

3.3 Degeneracy of $M^2 = 3$ states for neutrino and U quark

The degeneracies of fermionic $M^2 = 3$ states is evaluated for $N = 0, 1, 2, 3, 4$ sectors. The difference between U quark and neutrino emerges first time for $M^2 = 3$ states.

1. $N = 0$ sector

The operators creating $N = 0$ states are $O_5^4(14)$, $O_c^4(6)$, $O_c^3 O_5^1(4)$ and $O_c^2 O_5^2(4)$:$14 + 6 + 4 + 4 = 28$ altogether. The numbers of the color singlet

operators O_c^k, which satisfy gauge conditions for $L^1, G^{1/2}$ and possibly for L^2 can be easily found from the table of appendix F (table 8.3).

Consider next gauge conditions.

a) Gauge conditions for L^1, G^1, L^2 imply that the coefficients of operators O_5^4, $O_c^3 O_5^1$ and $O_c^2 O_5^2$ vanish. The gauge conditions for O_c^4 leave only single operator O_c^4 so that the contribution to degeneracy is $D(2,0) = 2$.

For U quark there exist no gauge invariant color singlet operators O_c^4 as the table of appendix F shows so that $D(2,0) = 0$ for U quark. There are no further differences between U quark and neutrino.

2. $N = 1$ sector

The following operators are present in $N = 1$ sector.

a) Single particle operators O_i^4 ($13 \cdot 4 = 52$).

b) Two particle operators $O_i^3 O_5^1 (8 \cdot 2 \cdot 4 = 4 \cdot 16)$, $O_i^2 O_5^2$ ($4 \cdot 4 \cdot 4 = 64$). $O_i^1 O_5^3$ ($2 \cdot 8 \cdot 4 = 64$): $4 \cdot 48$ altogether.

c) $O_c^3 O_i^1 (16)$ (these operators are eliminated by gauge conditions for $G^{1/2}$ and L^1), $O_c^2 O_i^2$ ($4 \cdot 4 = 16$), $O_c^2 O_5^1 O_i^1$ ($2 \cdot 2 \cdot 4 = 16$). The operators O_c^k satisfy the gauge conditions in color degrees of freedom except possibly the gauge condition for L^2. There are $4 \cdot 8$ operators altogether

The total number of operators is $4 \cdot (13 + 48 + 8) = 276$.

Gauge conditions for L^1 and G^1 correspond to $N = 1$ operators $O_i^3(104)$ and $N = 0$ operators $O^3(11)$: $104 + 11 = 115$ altogether. Gauge conditions for L^2 correspond to $N = 1$ operators O_i^2 (32) and $N = 0$ operators $O_5^2(4)$ and $O_c^2(1)$: 37 altogether. Supersymmetry implies doubling for of L^2 gauge conditions in $N = 0$ sector and one has 42 conditions. The total number of gauge conditions is $115 + 115 + 42 = 272$ and the total number of allowed operators is $276 - 272 = 4$ and one has $D(2,1) = 8$

3. $N = 2$ sector

The following operators are present in $N = 2$ sector.

a) The operators $O_i^2 O_j^2$ ($6 \cdot 4 \cdot 4 = 6 \cdot 16$).

b) The operators $O_i^3 O_j^1 (12 \cdot 8 \cdot 2 = 12 \cdot 16)$

c) The operators $O_i^2 O_j^1 O_5^1$ ($12 \cdot 4 \cdot 2 \cdot 2 = 12 \cdot 16$)

d) The operators $O_i^1 O_j^1 O_5^2$ ($6 \cdot 2 \cdot 2 \cdot 4 = 6 \cdot 16$)

e) $O_i^1 O_j^1 O_c^2$ ($6 \cdot 2 \cdot 2 = 24$)

The total number of states is $12 \cdot 35 = 426$.

Gauge conditions for L^1 and G^1 correspond to $N = 2$ operators $O_{ij}^3(144)$ and $N = 1$ operators $O_i^3(104)$. Gauge conditions for L^2 correspond to operators $O_{ij}^2(24)$ and $N = 1$ operators $O_i^2(32)$: super symmetry doubles the gauge conditions associated with $O^2(i)$. The total number of gauge conditions is $144 + 144 + 104 + 104 + 24 + 64 = 584$, which exceeds the number of the operators at hand so that the contribution to degeneracy is $D(2,2) = 0$.

4. $N = 3$ and $N = 4$ sectors

The operators creating states in $N = 3$ sector are:
a) $O_i^2 O_j^1 O_k^1$ ($4 \cdot 3 \cdot 4 \cdot 2 \cdot 2 = 12 \cdot 16$)
b) $O_i^1 O_j^1 O_k^1 O_5^1$ ($4 \cdot 16 = 64$).
The total number of operators is 16^2.

Gauge conditions for L^1 and G^1 correspond to the operators
i) $N = 3$ operators $O_i^1 O_j^1 O_k^1$ ($4 \cdot 8 = 32$).
ii) $N = 2$ operators $O_i^2 O_j^1$ ($12 \cdot 4 \cdot 2 = 8 \cdot 12$) and $O_i^1 O_j^1 O_5^1$ ($12 \cdot 2 \cdot 2 + 6 \cdot 2 \cdot 2 = 6 \cdot 12$).
The total number of conditions is $14 \cdot 12 + 32$.
Gauge conditions for L^2 correspond to operators $O_i^1 O_k^1$ ($12 \cdot 2 + 6 \cdot 2 = 3 \cdot 12$).
Total number of gauge conditions is $31 \cdot 12 + 64$ and larger than the number of operators (16^2) so that no solutions to gauge conditions are obtained. Situation is same in $N = 4$ sector and one has $D(2,3) = D(2,4) = 0$. The total degeneracy for neutrino is $D(2) = 10$. For U quark the total degeneacy is $D(2) = 8$.

4 Appendix B: Degeneracies for charged leptons and D type quarks

The calculation of $M^2(cm)$ for charged leptons is easy task since the operators creating $M^2 = 0$ and $M^2 = 3/2$ states have conformal weights $\Delta = 2$ and $\Delta = 1$. The operators O^1 creating massless states are linear combinations of 8 operators O_i^1 and gauge conditions for L^1 and G^1 reduce their number to 6, which gives $D(0) = 12$. From the calculation of neutrino masses one has $D(3/2) = 40$ and $D(3) = 80$. The table summarizes the results.

M^2	N=0	N=1	N=2	N=3	N=4	D
0	0	1	0	0	0	12
3/2	0	20	20	0	0	40
3	0	44	32	4	0	80

Table 4.1. The degeneracies of charged leptons and D type quarks with $M2 = 0, 3/2, 3$ in super selection sectors with $N = 0, 1, 2, 3, 4$. Last column gives total degeneracies.

5 Appendix C: Degeneracies for colored excitations of leptons and quarks

Before going to the results it is useful to notice that color super generators $F^{A1/2}$ are leptoquarks by the previous results. This means that the states created by operators with half odd integer Δ are in fact carriers of baryon number, that is quarks! This in turn implies that quarks would belong to triality zero representation and this is not in accordance with Quantum TGD. One manner to get out of difficulties is to assume strong form of G-parity rule: only $\Delta \in Z$ excitations are allowed and leptoquarks are not possible. One can also consider the possibility that for many lepton states only the entire state satisfies G-parity rule but composite leptons are allowed to exist in states of wrong G-parity.

The calculation of degeneracies for colored excitations reduces to the previous calculations for ordinary leptons:
a) The table 8.3 of appendix F about color Super Virasoro shows that only gauge invariant colored representations with conformal weight not larger than 2 are 10, $\overline{10}$ with conformal weight $n = 1$ and 27 with conformal weight $n = 2$. This means that all leptons allow color decuplet massless state and neutrinos also 27 dimensional massless state.
b) The degeneracies of corresponding states are easily evaluated. For decuplet the operators creating states with given mass squared have conformal weight one unit smaller than for ordinary leptons. For decuplet these operators have conformal weight two units smaller than for ordinary neutrino.
c) Also U type quarks allow massless color excitations as the table for multiplicities for gauge invariant color multiplets in Ramond representation shows.

The excitations correspond to 10, $\bar{1}0$ and have conformal weight $n = 2$ so that are possible for U quarks only.

d) The results imply that the previous calculations for leptons and quarks make possible to deduce the degeneracies of color excitations given in the table below.

fermion	D(0)	D(3/2)	D(3)	$M_R/m_e\sqrt{\frac{M_{127}}{p}}$
e^{10}	2	12	40	$\sqrt{\frac{9}{5+\frac{2}{3}}}$
ν^{10}	12	40	80	1
ν^{27}	2	12	40	$\sqrt{\frac{9}{5+\frac{2}{3}}}$
U^{27}	2	12	40	$\sqrt{\frac{9}{5+\frac{2}{3}}}$

Table 6.1. Degeneracies and masses for light color excitations for fermions. Note that 27-plet appears twice.

6   Appendix D: Detailed calculation of gauge boson masses

The results for bosonic degeneracies for nonexotic gauge bosons are summarized in the following table.

Boson	$M^2 = 0$	$M^2 = 3/2$
γ	3	2
Z^0	5	4
$W^{\pm}$	3	11

Table 6.1. The degeneracies of $M^2 = 0$ and $M^2 = 3/2$ states for electroweak gauge bosons.

The degeneracies of exotic bosons are listed in table below. The charge operator O associated with ground state characterizes the boson in question.

Type	O	$M^2 = 0$	$M^2 = 3/2$	M^2
$J = 0$	1	0		ah
	$F_5^{1/2}$	0		ah
	$F_k^{1/2}, k = 3, 4$	1	1	h
	$F_3^{1/2} F_4^{1/2}$	2	3	h
	$F_k^{1/2} F_5^{1/2}, k = 3, 4$	1	2	$\frac{3}{p}$
	$F_3^{1/2} F_4^{1/2} F_5^{1/2}$	2	10	h
$(J = 0, 8)$	1	0		ah
	$F_k^{1/2}, k = 3, 4$	0		ah
	$F_3^{1/2} F_4^{1/2}$	0		ah
	$F_3^{1/2} F_4^{1/2} F_5^{1/2}$	3	4	$\frac{2}{p}$
$J = 1$	$E = p^k \gamma_k^{1/2}$	1	1	h
$(J = 1, 8)$	$E F_k^{1/2}, k = 3, 4$	0		ah
	$E F_3^{1/2} F_4^{1/2}$	3	1	h
	$E F_k^{1/2} F_5^{1/2}$	3	1	h
	$E F_3^{1/2} F_4^{1/2} F_5^{1/2}$	1	5	h
$(J = 1, 10/\bar{10})$	$E F_k^{1/2} F_l^{1/2}, k \neq l = 3, 4, 5$	1	2	$\frac{3}{p}$
$(J = 0, 10/\bar{10})$	$F_3^{1/2} F_4^{1/2} F_5^{1/2}$	1	2	$\frac{3}{p}$
$(J = 0, 10/\bar{10})$	$G_5^{3/2}$	1	0	0
$(J = 1, 27)$	$E F_k^{1/2}, k = 3, 4, 5$	1	1	h
$(J = 0, 27)$	$F_k^{1/2} F_l^{1/2}, k \neq l = 3, 4, 5$	1	1	h
$(J = 1, 27)$	E	1	0	0
$(J = 0, 27)$	$F_k^{1/2}, k = 3, 4, 5$	1	0	0

Table 6.2. Degeneracies of $M^2 = 0, 3/2$ and (in some cases) $M^2 = 3$ states for exotic bosons. Exotic bosons are characterized by operator O creating the (in general tachyonic) ground state for the boson. The masses are given for $T = 1$. 'ah' (' absolutely heavy') means that state has Planck mass independentely of the value of p and T and 'h' (heavy ') means that state has Planck mass for primes, which are not too large and for $T = 1$: for $T = 1/2$ state has mass of order $1/p$. No massless exotic noncolored bosons are predicted. There are however some massless colored states. Note

that $F_k^{1/2}$ contains isospin index and the case of intermediate gauge bosons suggests certain constraints on isospin indices.

6.1 Photon

Photon and Z^0 differ in one respect only: the charge matrix for photon acts in the sectors $k = 3, 5$ and for Z^0 in sectors $k = 3, 4, 5$. Photon and Z^0 are superpositions of states k is 'active' that is N-S vacuum with $h_k = 1/2$ is excited. The 3 sectors decouple from each other and apart from the presence of nonzero norm states created by L_5^2 and $G^k, k > 3/2$ from $h_5 = 0$ vacuum the sectors are essentially identical.

The ground state for photon is given by the following expression

$$|vac\rangle_\gamma = E(aF_3^{1/2,3} + bF_5^{1/2})|vac\rangle \tag{20}$$

where the values of the coefficients a and b are such that the coupling $Q_{em} = I_3 + Q_K/2$ to fermions results. State satisfies gauge conditions. For definitess it will be assumed that polarization operator acts in sector $i = 2$. Note that $T0_3$ and T_5^0 and also their super counterparts appearing in the state measure vectorial isospin and Kähler charge.

1. Massless states

Massless photon state is obtained by applying operators $O^{3/2}$ to the ground state. Polarization operator P is not allowed in the construction. State contains two terms of same form corresponding to $k = 3, 5$ charge operators and these terms do not couple to each other in gauge conditions. Therefore one can consider only the second term. The list of these operators for say $k = 3$ case is following:
a) Single particle operators $O_i^{3/2}, i = 2, 3$: $2 + 2 = 4$ altogether.
b) Two particle operators $O_i^1 O_j^{1/2}$, $i \neq j = 2, 3$ (2).
c) $G_5^{3/2}$ gives one additional operator in $k = 3$ case. Gauge conditions are identically satisfied for this operator.
There are 7 (6) $O^{3/2}$ operators altogether for $k = 3$ ($k = 5$).

Gauge conditions for $G^{1/2}$ in case of $k = 3$ correspond to operators O^1 given by O_i^1, $i = 2, 3$ (2) and $O_2^{1/2} O_3^{1/2}$: 3 altogether. For L^1 gauge conditions

correspond to the 2 operators $O^{1/2}$ given by $O_i^{1/2}$, $i = 1, 2$. Gauge conditions for L^2 are identically satisfied. This gives 5 conditions altogether so that $k = 3$ gives $D(0, 3) = 2$ states and $k = 5$ $D(0, 5) = 1$ state. Ground state degeneracy for photon is $D(0, \gamma) = 2 + 1 = 3$.

If the inner product is modified then 2 additional operators $O_i^{3/2}$, $i = 1, 4$ are allowed in $N = 1$ sector for $k = 3$. For $k = 5$ therere are 3 operators. Gauge condition associated with these operators are trivially satisfied so that 5 states are obtained. Ground state degeneracy becomes $D(0, \gamma) = 3 + 5 = 8$.

2. $M^2 = 3/2$ states

Massive states at level $\Delta = 1$ are created by operators $O^{5/2}$ from ground states. Again conditions are independent for $k = 3$ and $k = 5$ contributions to the charge operator. For $k = 3$ the list of excitations not involving color is following:

a) Single particle operators $O_i^{5/2}$, $i = 2, 3$ (8).

b) Two-particle operators $O_i^2 O_j^{1/2}$ (3+3 = 6), $O_i^{3/2} O_j^1$ (2+2 = 4), $i \neq j = 2, 3$: 10 altogether.

c) The 6 operators $G_5^{5/2}$, $G_i^{1/2} L_5^2$ and $L_i^1 G_5^{3/2}$ and $G_5^{3/2} G_2^{1/2} G_3^{1/2}$, $i = 2, 3$ acting in $u(1)$ degrees of freedom are present only for $k = 3$.

d) The operators $O_c^{5/2}$ (1, see for table 8.3 in appendix F) and $O_c^2 G_i^{1/2}$ (2), $i = 1, 2$ acting in color degrees of freedom. There are 3 operators altogether. The total number of operators is $8 + 10 + 6 + 3 = 27$ for $k = 3$ and 21 for $k = 5$.

a) Gauge conditions for $G^{1/2}$ ($k = 3$) correspond to the operators O^2 given by

i) O_i^2, $i = 2, 3$ (3 + 3 = 6)

ii) $O_i^{3/2} O_j^{1/2}$, $O_i^1 O_j^1$ (5), $i \neq j = 2, 3$.

ii) L_5^2, $G_5^{3/2} G_i^{1/2}$, $i = 2, 3$ present only for $k = 3$ (3).

iii) $O_c^2 = F^{A3/2} F^{A1/2}$. (1)

There are $6 + 5 + 3 + 1 = 15$ conditions for $k = 3$ and 12 conditions for $k = 5$.

b) For L^1 the gauge conditions correspond to the operators $O^{3/2}$ and their number is 7 for $k = 3$ and 6 for $k = 5$.

c) For $G^{3/2}$ gauge conditions correspond to the 2 operators O_i^1, $i = 2, 3$. For L^2 gauge conditions correspond to unit matrix.

The total number of gauge conditions is $15 + 7 + 2 + 1 = 25$ for $k = 3$

and $12 + 6 + 2 + 1 = 21$ for $k = 5$. The contribution to degeneracy is $D(1,3) = 27 - 25 = 2$ for $k = 3$ and $D(1,5) = 21 - 21 = 0$ for $k = 5$ so that the degeneracy is $D(1,\gamma) = 2$. Since $D(0,\gamma) = 3$ photon is essentially massless provided $k(B)$ is multiple $3/2$.

In $N = 1$ there are operators $G_i^{5/2}$, $L_i^2 O_j^{1/2}$, $G_i^{3/2} O_j^1$, $G_i^{3/2} O_2^{1/2} O_3^{1/2}$ $j = 2,3$, $i = 1,4$ for $k = 3$: their total number is 11. Gauge conditions for $G^{1/2}$ correspond to 6 operators L_i^2 , , $G_i^{3/2} O_j^{1/2}$, $j = 2,3$, $i = 1,4$ for $k = 3$. For L^1 and L^2 gauge conditions correspond to the operators $G_i^{3/2}$ (2) and $O_j^{1/2}, j = 2,3$ (2). The total number of gauge conditions is 10 so that one obtains one state for $k = 3$. For $k = 5$ the number of states is 2. This implies $D(3/2)/D(0) = (2+3)/8 \neq 1$.

The conclusion is that photon possesses negligibly small mass for $T(ew) = 1/2$, $k \geq 2$ provided the value of the parameter $k(B)$ is $k(B) = 3/4$. At level M_{127} this mass would be of order $m_e/\sqrt{M_{127}} \simeq 10^{-13}$ eV. The results apply almost as such to the case of Z boson.

6.2 Z^0 boson

The ground state for Z^0 is given by the following expression

$$|vac\rangle_Z = E(aF_3^{1/2,3} + bF_4^{1/2} + cF_5^{1/2})|vac\rangle$$

(21)

where the coefficients are determined from the requirement that Z^0 couplings to are given by $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$. What differentiates between Z^0 and γ is the fact that the coupling is not purely vectorial for Z^0 and three active sectors $k = 3, 4, 5$ become possible. The degeracies for various states can be obtained using the results for photon. The total degeneracy for level n is given by

$$D(n) = \sum_{k=3,4,5} D(n,k) = 2D(n,3) + D(n,5)$$

(22)

The degeneracies $D(n,3) = D(n,4)$ and $D(n,5)$ have been already calculated:

$$D(0,3) = 2, \quad D(0,5) = 1$$
$$D(1,3) = 2, \quad D(1,5) = 0$$

(23)

so that one has for the degeneracies of various states

$$D(0,Z) = 5$$
$$D(1,Z) = 4$$

(24)

The ratio $s(Z) = k(B)D(1,Z)/D(0,Z) = n6/5$ ($k(B) = n3/2$ from the masslessness of photon) implies that Z^0 must correspond to temperature $T(Z) = 1/2$. $n = 1$ turns out to be the only physically interesting values of n. A small calculation shows that the real counterpart of $p^2/5(n = 4)$ is $4/5$, which corresponds to $s(Z) = 0$ and $X = 12$ in small quantum number approximation.

6.3 W boson

The ground state for W boson is given by the following expressions

$$|vac\rangle_W = E \sum_{i,j=1,2} q_{ij} F_3^{1/2,i} F_4^{1/2,j} |vac\rangle$$

(25)

The coefficients q_{ij} are determined by the charge matrix of W. The difference between Z^0 and W is that the sectors $i = 3, 4$ are both active simultaneously.

Massless states are created by applying operators O^1 to the W ground state. The list of O^1 operators is following.
a) Single particle operators O_i^1, $i = 2, 3, 4$: 3 altogether.
b) Two particle operators $O_i^{1/2} O_j^{1/2}$, $i \neq j = 2, 3, 4$: 3 altogether.
The total number of operators is $3 + 3 = 6$. $G^{1/2}$ gives 3 gauge conditions (operators $O_i^{1/2}$) . L^1 gives one gauge condition, which is however not an independent one due to the antisymmetry of the coefficient matrix associated with $O_i^{1/2} O_j^{1/2}$. Therefore ground state degeneracy is $D(0, W) = 6 - 3 = 3$.

$M^2 = 3/2$ states are created by applying operators O^2 to the W ground state. The list of O^2 operators is following.
a) Single particle operators O_i^2, $i = 2, 3, 4$: 9 altogether.
b) Two particle operators $O_i^{3/2}O_j^{1/2}$ (12), $O_i^1 O_j^1$ (3), $i \neq j = 2, 3, 4$: 15 altogether.
c) 3-particle operators $O_i^1 O_j^{1/2} O_k^{1/2}$, $i \neq j \neq k = 2, 3, 4$: 3 altogether.
d) Operators $L_5^2, G_5^{3/2} O_i^{1/2}, i = 2, 3, 4$ acting in $u(1)$ degrees of freedom: 4 altogether.
e) Operator O_c^2 acting in color degrees of freedom.
The total number of operators is $9 + 15 + 3 + 4 + 1 = 32$.

Consider next gauge conditions.
a) $G^{1/2}$ gauge conditions correspond to
i) single particle operators $O_i^{3/2}$, $i = 2, 3, 4$: 6 altogether.
ii) two-particle operators $O_i^1 O_j^{1/2}$, $i \neq j = 2, 3, 4$: 6 altogether.
iii) 3-particle operator $O_1^{1/2} O_2^{1/2} O_3^{1/2}$.
iv) operator $G_5^{3/2}$.
There are $6 + 6 + 1 + 1 = 14$ operators altogether.
b)L^1 gauge conditions correspond
i) single particle operators O_i^1, $i = 2, 3, 4$: 3 altogether.
ii) two particle operators $O_i^{1/2} O_j^{1/2}$, $i \neq j = 2, 3, 4$: 3 altogether.
$3 + 3 = 6$ conditions altogether.
c) L^2 gives one condition.
The total number of gauge conditions is $14 + 6 + 1 = 21$ and the degeneracy is $D(1, W) = 32 - 21 = 11$. W mass is is of order Planck mass unless one assumes $T(W) = 1/2$. In this case second order contribution is $n11/2$ and one has $s(W) = 0$ and $X(W) = 8$ for $n = 1$.

The value of Weinberg angle serves as test for the physicality of the scenario. For $n = 1$ $(k(B) = 3/2)$ one obtains

$$ sin^2(\theta_W) \quad = \quad 1 - \frac{M_W^2}{M_Z^2} = \frac{3}{8} \tag{26} $$

Encouragingly, the value is typical value of the parameter in the symmetry limit in GUTs. Electron gauge boson mass ratio comes out correctly if one assumes $k(F) = 2$.

6.4 Gluon

There are no color octet operators O_c^{kA} for $k = 1, 3/2, ..., 4$ satisfying the gauge conditions associated with L^1, $G^{1/2}$ and L^2 as the study of the table (see 8.3 in appendix F) shows. This means that a natural identification for the gluon ground state is as the state created by the operator

$$
\begin{aligned}
G &= EF^{A1/2} \\
E &= \epsilon_k \gamma_{1/2}^k
\end{aligned}
\tag{27}
$$

The interesting physical states are created by operating with $so.. \times u(1)$ operators on this state. Massless states are created by operating with the operators $O_i^{3/2}$, $i = 2, 5$ to this state. Gauge conditions for $G^{1/2}$ and L^1 leave only $G_5^{3/2}$ so that the ground state degeneracy is $D(0, G) = 1$.

For $M^2 = 1$ state the 7 operators $O^{5/2}$ are given by $O_2^{5/2}$, $G^{5/2}$, $L_5^2 G_2^{1/2}$ and $G_5^{3/2} L_2^1$. The number of gauge conditions is 9 corresponding to operators O_2^2, L_5^2, $G_5^{3/2} F_2^{1/2}$ (5) associated with $G^{1/2}$, to the operators $O_2^{3/2}$, $G_5^{3/2}$ (3) corresponding to L^1 and operator O_2^1 corresponding to L^2. There are no solutions to gauge conditions so that gluon is exactly massless in order $O(p)$. Since $D(0, G) = 1$ the next order gives negligible contribution to mass.

7 Appendix E: Exotic states

There are many candidates for exotic particles and one must show that these states
a) do not allow massless ground state satisfying gauge conditions or
b) (assuming that $T = 1$) possess Planck mass ($D(3/2)/2D(0)$ is not an integer) or
c) have nonvanishing $D(1)$ or at least $X \bmod 2D(0) \neq 0$, $X = 3(2D(3) - D(3/2)^2/D(0))$ so that they become massive ($k(B) = 3/2$ is assumed in the formula).

Exotic states can be classified according to the properties of the ground state associated with the particle.
a) Exotic scalars with ground state, which can have nonvanishing electroweak quantum numbers.

b) Scalar gluons.
c) Exotic spin 1 electroweak bosons
d) Exotic spin 1 gluons.

7.1   Exotic scalars

It turns out that there are many exotic scalars allowing massless excitation but that for $T = 1$ no exotic scalar remains massless in Higgs mechanism. For $T = 1/2$ there are two massless scalars. These states are created by the following operators.

a) $O = 1$: Planck mass
Massless states are created by operators $G^{5/2}$ and $O_c^{5/2}(1)$. Gauge conditions allow no massless excitations.

b) $O = F_5^{1/2}$: Planck mass
There are no operators creating massless states and state possesses Planck mass.

c) $O = F_k^{1/2}$, $k = 3, 4$: Planck mass for $T = 1$.
The operators O^2 creating massless states are
i) single particle $O_k^2(3)$ and operators $L_5^2(1)$, $G_5^{3/2}G_k^{1/2}(1)$
ii) the operator $O_c^2(1)$ creatig color excitation.
There are 6 operators altogether.

c1) Gauge conditions of $G^{1/2}$ correspond to single particle operators $O_k^{3/2}(2)$ and operator $G_5^{3/2}$: 3 altogether.
c2) Gauge conditions L^1 correspond to operator $O_k^1(1)$
c) Gauge conditions for L^2 corresponds to unit operator.
Total number of gauge conditions is 5 so that $D(0) = 1$ results. The states can be regarded as scalar counterparts of axial and vectorial gauge bosons with $I_3 = 0$. $D(0) = 1$ implies that these have Planck mass for $T = 1$ and $k(B) = 3/2$ if $D(3/2)$ is odd. It turns out that $D(3/2) = 1$!

Because of its central importance the result $D(3/2) = 1$ will be derived by considering opertors O^3 creating $\Delta = 1$ states. Consider first O^3 operators creating $M^2 = 3/2$ states. There are
a) single particle operators $O_k^3(5)$ and operators L_5^3, $G_5^{5/2}G_k^{1/2}(1)$, $L_5^2O_k^1(1)$ $G_5^{3/2}O_k^{3/2}(2)$: 10 altogether.

b) The operators $O_c^3(2)$ $O_c^{5/2}O_k^{1/2}(1)$, $O_c^2O_k^1(1)$ creating color excitations. There are 4 operators altogether.

Gauge conditions allow no operators of type a):
a) Gauge conditions of $G^{1/2}$ correspond to single particle operators $O_k^{5/2}(4)$ and operators $G_5^{5/2}$, $L_5^2G_k^{1/2}(1)$, $G_5^{3/2}O_k^1(1)$: 7 altogether.
b) Gauge conditions L^1 correspond to operators O^2: their total number is 5
c) Gauge conditions for L^2 correspond to operator L_k^1.
Total number of gauge conditions is $7+5+1 = 13 > 10$ so that no solutions to gauge conditions are possible.
For operators of type b) gauge conditions for $G^{1/2}$ correspond to operators $O_c^{5/2}(1)$ and $O_c^2O_k^{1/2}(1)$. Gauge conditions for L^1 corresponds to the operators O_c^2. L^2 gives no conditions. The number of gauge conditions is therefore 3 and just one operator satisfying gauge conditions remains and one has $D(3/2) = 1$.

c) $O = F_k^{1/2}F_l^{1/2}$, $k, l = 3, 4, 5$.
$(k, l) = (3, 4)$: Planck mass for $T = 1$.
$(k, l) = (3, 5), (4, 5)$: $\frac{3}{p}$ for $T = 1$.
For $k = 3, l = 4$ the ground state degeneracy is $D(0) = 2$ and for $(k, l) = (3, 5), (4, 5)$ the ground state degeneracy is $D(0) = 1$. These states can be regarded as scalar partners of W and as a $u(1)$ boson coupling to the product of axial/vectorial isospin and Kähler charge. The value of $D(3/2)$ is 3 for $(k, l) = (3, 4)$ so that this state has Planck mass. $D(3/2) = 2$ for $l = 5$ boson implies that this state has mass doesn't produce troubles if it condenses on level with small p. For $T = 1/2$ the state is essentially massless.

d) $O = F_3^{1/2}F_4^{1/2}F_5^{1/2}$: Planck mass for $T = 1$.
Massless ground state degeneracy is $D(0) = 2$. State couples to the product of $I^{\pm}$ and Kähler charge. $D(3/2) = 10$ implies that state has Planck mass for $T = 1$ and mass $M^2 = \frac{1}{2p}$ for $T = 1/2$.

7.2 Exotic scalar gluons

There is only one exotic scalar gluon.
a) $O = F^{A1/2}$, $F_k^{1/2}F^{A1/2}$ and $F_3^{1/2}F_4^{1/2}F^{A1/2}$ allow no massless states and possess therefore Planck mass.

b) $O = F_3^{1/2} F_4^{1/2} F_5^{1/2} F^{1/2A}$: $M^2 = \frac{2}{p}$ for $T = 1$.
There are $D(0) = 3$ massless states. State couples to $I^{\pm} Q_K T^A$. $D(3/2) = 4$ implies that state is not massless for $T = 1$ although it is 'light': for $T = 1/2$ state would be massless.

7.3 Exotic spin one color singlet bosons

The operator creating these states is of form $B = EO$.

$O = 1$: Planck mass for $T = 1$.
This boson couples to fermion number. Massless ground state has $D(0) = 1$ $D(3/2) = 1$ follows using the same derivation as applied in case of scalar bosons $(O = F_k^{1/2})$ so that state has Planck mass for $k(B) = 3/2$ and $T = 1$.

7.4 Exotic spin one gluons

Again states are of form $B = EO$.
a) $O = F_k^{1/2} F^{A1/2}$: Planck mass.
Gauge conditions allow no massless ground states.

b) $O = F_k^{1/2} F_l^{1/2} F^{A1/2}$, $k, l = 3, 4, 5$: Planck mass for $T = 1$.
Massless ground state degeneracy is $D(0) = 3$. These states include color octet counterpart of W boson. These states couple to $I^{\pm} T^A$ or to $I_{A/V}^3 Q_K$. $D(3/2) = 1$ makes these states very massive for $k(B) = 3/2$ and $T = 1$.

c) $O = F_3^{1/2} F_4^{1/2} F_5^{1/2} F^{A1/2}$: Planck mass for $T = 1$.
Massless ground state degeneracy is $D(0) = 1$. $D(3/2) = 5$ implies that this state has Planck mass for $k(B) = 3/2$ and $T = 1$.

7.5 Higher color representations

The table of appendix F for gauge invariant N-S type color operators with various conformal weights n shows the existence of $n = 1$ decuplets 10 and $\overline{10}$, $n = 3/2$ 27-plet and doubly degenerate $n = 2$ 27-plet. This implies the existence of exotic colored bosons. Also p-adic temperature $T = 1/2$ is possible for these states since the existence of very light colored states does not imply new long range forces.

a) Decuplet operators creating massless states have the general form $O = O^{3/2}O_c^{10,1}$, where $O^{3/2}$ acts in $so... \times u(1)$ degrees of freedom. There are the following possibilities:

i) For $O^{3/2} = EF_k^{1/2}F_l^{1/2}$, $k \neq l = 3,4,5$ and $O^{3/2} = F_3^{1/2}F_4^{1/2}F_5^{1/2}$ gauge conditions are identically satisfied and one has $D(0) = 1$. $M^2 = 3/2$ excitations are created by the 3 operators $L_i^1, i = 2, k, l$ (3,4,5) and L^1 gauge condition implies that degeneracy of $M^2 = 3/2$ states is $D(3/2) = 2$ so that for $T = 1$ state has mass $M^2 = \frac{3}{p}$. For $T = 1/2$ the state is essentially massless.

ii) $O^{3/2} = G_5^{3/2}$. Gauge conditions are identically satisfied. There are neither $M^2 = 3/2$ nor $M^2 = 3$ excitations satisfying gauge conditions so that these states are exactly massless.

iv) There are also operators of form $O^{3/2} = O^{n_1}O^{n_2}$, $n_1 + n_2 = 3/2$, where O^{n_1} is Super Virasoro generator and O^{n_2} is constructed from the operators $O_i^{1/2}$ but these operators give no states satisfying gauge conditions.

b) The operators creating massless states in case of $O^{27,3/2}$ have the general form $O = O^1 O_c^{27,3/2}$. Only $O^1 = EF_k^{1/2}$, $k = 3,4,5$ and $O^1 = F_k^{1/2}F_l^{1/2}$ give states satisfying gauge conditions and degeneracies are $D(0) = 1$ and $D(3/2) = 1$. These states have Planck mass for $T = 1$ and for $T = 1/2$ the mass is $M^2 = \frac{1}{2p}$.

c) The operators creating massless states in case of $O^{27,2}$ have the general form $O = O^{1/2}O_c^{27,2}$ with $O^{1/2} = E$ or $O^{1/2} = F_k^{1/2}$, $k = 3,4,5$ and there exists double-fold degeneracy. The degeneracies are $D(0) = 1$ and $D(3/2) = 0$ and the states are massless for both $T = 1$ and $T = \frac{1}{2}$.

8 Appendix F: Construction of positive norm states in color degrees of freedom

The construction of positive norm states for various values of conformal weight is essential ingredient in the calculation of degeneracies for various values of mass squared operator in order to estimate thermal mass expectation value:

i) If one has obtained the multiplicities of various representations with weight n then it is easy to calculate the multiplicities for gauge invariant states. If

gauge conditions associated with $G^{1/2}$, L^1 and L^2 in N-S representation induce surjective maps to the levels $n-1/2$, $n-1$ and $n-2$ then the multiplicity of gauge invariant representation is given by $m = m(n) - m(n-1/2) - m(n-1) - m(n-2)$.

ii) For states involving tensor product of several Super Virasoro representations (say, the representation of baryon as 3-quark state) it is straightforward task to form tensor product in color degrees of freedom if multiplicities of nonzero norm states are known and apply then gauge conditions to the total Super Virasoro.

The construction of nonzero norm states relies on the following observations.

a) The states at each level n (conformal weight) of color Kac Moody algebra can be classified into irreducible representations of color group. The states are created by polynomials $O(F)$, where F is short hand notation for the super generators F^{Ak} of color Kac Moody algebra. Bosonic generators are not used since they are expressible in terms of F^{Ak} and their use would lead to double counting problems since T^{An} is expressible as bilinear of F.

b) Zero norm states must be eliminated. They are created by product operators of form

$$
\begin{aligned}
O &= O_1 O_2 \\
O_1 &= O(F) \\
O_2 &= O_2(L, G)
\end{aligned}
$$

(28)

O_2 is operator formed from Super Virasoro generators and creates zero norm state ($c = h = 0$). O_1 is polynomial of fermionic generators F^{Ak} in color Kac Moody algebra.

c) The operators O_2 are constructed from Super Virasoro generators, which do not annihilate vacuum state. For N-S algebra there are generators, which annihilate vacuum automatically and must be excluded from construction. The generators $G^{1/2}$ and L^1 are proportional to T^{A0} and annihilate therefore vacuum. The generator L^2 reduces essentially to $T^{1A}T^{1A}$, when acting on vacuum. The representation $T^{1A} \propto f^{ABC}F^{1/2B}F^{1/2C}$ together with Jacobi identities demonstrates that the action of $T^{1A}T^{1A}$ to vacuum actually vanishes. Since L^1 and L^2 generate Virasoro algebra all generators L^n annihilate

45

vacuum so that only the operators constructed from $G^k, k > 1/2$ remain to be considered. For Ramond representations the entire Super Virasoro algebra must be taken into consideration since T^{A0} do not annihilate ground state triplet.

d) The operators O_1 creating nonzero norm sates can be classified into irreducible representations of color group. The basic building blocks are the representations defined by N:th order monomials of generators F^{Ak} with k fixed. These representations are completely antisymmetrized tensor products of $N = 0, 1, ..., 8$ octets and representation content is same for all values of k. The representation content can be coded into multiplicity vector $m(N; k)$, $k = 1, 8, 10, ...,$

e) Once the representation contents for antisymmetrized tensor products are known in terms of multiplicity vectors, the representation contents for tensor products of N_1, k_1 and N_2, k_2 can be determined by standard tensor product construction since anticommutativity does not produce no effects for $k_1 \neq k_2$. One can express the multiplicity vector for the tensor product $(N_1, k_1) \otimes (N_2, k_2)$ in terms of the multiplicity vector $D(k_1, k_2, k_3)$ for the tensor product of irreducible representations $k_1, k_2 = 1, 8, 10, ...$

$$m((N_1, k_1) \otimes (N_2, k_2; k) \quad = \quad m(N_1; k_1)D(k_1, k_2, k_3)m(N_2; k_2) \qquad (29)$$

f) It is useful to caculate total multiplicity vector $m(n; k)$ for each conformal weight n by considering all possible states having this conformal weight. The multiplicity vector is just the sum of multiplicity vectors of various tensor products satisfying $\sum N_i k_i = N$:

$$
\begin{aligned}
m(n; k) &= \sum_{S=N} m((N_1, k_1) \otimes .... \otimes (N_r, k_r; k)) \\
S &\equiv \sum N_i k_i
\end{aligned}
\qquad (30)
$$

The multiplicity vectors $m(n; k)$ are basic objects in the systematic construction of tensor products of several Super Virasoro algebras (say, in construction of many quark states).

8.1 Multiplicity vectors for antisymmetric tensor products

Consider first the construction of N-fold antisymmetric tensor products of octets F^{Ak}, k fixed. The tensor products are obviously analogous to the antisymmetric tensors of 8-dimensional space. The completely antisymmetric 8-dimensional permutation symbol $\epsilon_{A_1,....,A_8}$ transforms as color singlet and induces duality operation in the set of antisymmetric representations: the antisymmetric representations N are mapped to representations $8 - N$. This imlies that the representation contents are same for $N = 0$ and 8, $N = 1$ and 7, $N = 2$ and $N = 6$, $N = 3$ and $N = 5$ respectively. $N = 4$ is self dual. It is relatively easy to determine the representation content of the lowest completely antisymmetric representations and the results can be summarized conveniently as multiplicity vectors defined as

$$\bar{m} \equiv (m(1), m(8), m(10), m(\bar{10}), m(27), m(28), m(\bar{28}), m(64), m(81), m(\bar{81}), m(125), ...) \tag{31}$$

The multiplicity vectors are given by the following formulas

$$\begin{aligned}
\bar{m}(F) = \bar{m}(F^7) &= (0, 1) \\
\bar{m}(F^2) = \bar{m}(F^6) &= (0, 1, 1, 1) \\
\bar{m}(F^3) = \bar{m}(F^5) &= (1, 1, 1, 1, 1) \\
\bar{m}(F^4) &= (0, 2, 0, 0, 2))
\end{aligned} \tag{32}$$

where F^N denotes N:th tensor power of F^{Ak}.

8.2 Multiplicity vectors for various conformal weights for color Super Virasoro algebra

The next task is to calculate multiplicity vectors for various conformal weights. The task is straightforward application of Young Tableaux . The representation contents for various conformal weights for N-S algebra are given by

$$n = 0 : 1$$

$$n = 1/2 : 1/2$$
$$n = 1 : (1/2)^2)$$
$$n = 3/2 : 3/2 \oplus (1/2)^3$$
$$n = 2 : (3/2) \otimes (1/2) \oplus (1/2)^4$$
$$n = 5/2 : 5/2 \oplus (3/2) \otimes (1/2)^2 \oplus (1/2)^5$$
$$n = 3 : (5/2) \otimes (1/2) \oplus (3/2)^2 \oplus (3/2) \otimes (1/2)^3 \oplus (1/2)^6$$
$$n = 7/2 : 7/2 \oplus (5/2) \otimes (1/2)^2 \oplus (3/2)^2 \otimes (1/2) \oplus 3/2 \otimes (1/2)^4 \oplus (1/2)^7$$
$$n = 4 : (7/2) \otimes (1/2) \oplus (5/2) \otimes (3/2) \oplus (5/2) \otimes (1/2)^3 \oplus (3/2)^2 \otimes (1/2)^2 ...$$
$$\oplus (3/2) \otimes (1/2)^5 \oplus (1/2)^8$$
$$n = 9/2 : 9/2 \oplus (7/2) \otimes (1/2)^2 ...$$
$$\oplus (5/2) \otimes (3/2) \otimes (1/2) \oplus 5/2 \otimes (1/2)^4 \oplus (3/2)^3 ....$$
$$\oplus (3/2)^2 \otimes (1/2)^3 \oplus (3/2) \otimes (1/2)^6$$

$$(33)$$

Multiplicity vectors obtained as sums of multiplicity vectors associated with summands in the direct sum composition and are given by the following table

n	1	8	10	10	27	28	28	35	35	64	81	81
0	1											
1/2		1										
1		1	1	1								
3/2	1	2	1	1	1							
2	1	4	1	1	3							
5/2	2	6	3	3	4							
3	2	10	6	6	6			2	2	1		
7/2	4	16	8	8	12			4	4	2		
4	8	24	12	12	21	1	1	7	7	4		
9/2	10	36	21	21	32	1	1	12	12	8	1	1

Table 8.1. Multiplicity vectors for various conformal weights for N-S type Super Virasoro algebra.

Similar arguments can be used to deduce the multiplicity vectors in case of Ramond type Super Virasoro algebra.

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	80	$\overline{80}$	81	$\overline{81}$	125
0	1														
1		1													
2		1	1	1											
3	2	4	2	2	2										
4	2	10	4	4	6			1	1						
5	6	20	10	10	14			4	4	1					
6	12	40	22	22	32	1	1	10	10	6					
7	17	68	36	36	55	1	1	20	20	11			1	1	
8	33	124	70	70	113	5	5	44	44	29			5	5	1
9	70	276	170	170	276	16	16	122	122	94	1	1	22	22	6

Table 8.2. Multiplicity vectors for various conformal weights for Ramond type Super Virasoro algebra.

8.3 Elimination of zero norm state from N-S and Ramond algebra

Multiplicity vectors for the zero norm representations in N-S algebra can be constructed easily. There are two important point to notice.

i) The operators $G^{1/2}$, L^n annihilate vacuum and cannot be used in the construction of module of zero norm states.

ii) The polynomial O_1 of color Kac Moody super generators multiplying Virasoro operator O_2 must have nonvanishing norm in order to avoid double counting.

ii) The operators are most conveniently constructed by starting from $n = 0$ level and proceeding iteratively counting simultaneously the multiplicity vectors for nonzero norm states.

The list is of Super Virasoro operators generating zero norm states and elimination procedure is described in following:

$$
\begin{aligned}
n &= 3/2 : G^{3/2} \\
\bar{m}(3/2) &\rightarrow \bar{m}(3/2) - \bar{m}(0) \\
n &= 2 : G^{3/2} \\
\bar{m}(2) &\rightarrow \bar{m}(2) - \bar{m}(1/2)
\end{aligned}
$$

$$
\begin{aligned}
n &= 5/2 : G^{5/2}, G^{3/2} \\
\bar{m}(5/2) &\rightarrow \bar{m}(5/2) - \bar{m}(0) - \bar{m}(1) \\
n &= 3 : G^{5/2}, G^{3/2} \\
\bar{m}(3) &\rightarrow \bar{m}(3) - \bar{m}(1/2) - \bar{m}(3/2) \\
n &= 7/2 : G^{7/2}, G^{5/2}, G^{3/2} \\
\bar{m}(7/2) &\rightarrow \bar{m}(7/2) - \bar{m}(0) - \bar{m}(1) - \bar{m}(2) \\
n &= 4 : G^{5/2}G^{3/2}, G^{7/2}, G^{5/2}, G^{3/2} \\
\bar{m}(4) &\rightarrow \bar{m}(4) - \bar{m}(0) - \bar{m}(1/2) - \bar{m}(3/2) - \bar{m}(5/2) \\
n &= 9/2 : G^{9/2}, G^{5/2}G^{3/2}, G^{7/2}, G^{5/2}, G^{3/2} \\
\bar{m}(9/2) &\rightarrow \bar{m}(9/2) - \bar{m}(0) - \bar{m}(1/2) - \bar{m}(1) - \bar{m}(2) - \bar{m}(3)
\end{aligned}
\tag{34}
$$

The table of multiplity vectors for nonzero norm states reads as:

n	1	8	10	$\overline{10}$	27	28	$\overline{28}$	35	$\overline{35}$	64	81	$\overline{81}$
0	1											
1/2		1										
1		1	1	1								
3/2		2	1	1	1							
2	1	3	1	1	3							
5/2	1	5	2	2	4							
3	2	7	5	5	5			2	2	1		
7/2	2	12	6	6	9			4	4	2		
4	6	16	9	9	16	1	1	6	6	4		
9/2	6	24	14	14	24	1	1	10	10	7	1	1

Table 8.3. Table of multiplicity vectors for nonzero norm N-S type color representations for various values of the conformal weight n.

This table contains all essential as regards to the construction of gauge invariant states. Assuming surjectivity for the maps induced by the action of $G^{1/2}$, L^1 and L^2 one has general expression for the multiplicity vector of gauge invariant N-S type representations as

$$
\bar{m}(n)_{GI} = \bar{m}(n) - \bar{m}(n-1/2) - \bar{m}(n-1) - \bar{m}(n-2)
\tag{35}
$$

If some of the resulting multiplicities becomes negative it must obviously be replaced with zero. The analogous formula for Ramond representation is

$$\bar{m}(n)_{GI} \;=\; \bar{m}(n) - 2\bar{m}(n-1) - \bar{m}(n-1) - \bar{m}(n-2) \qquad (36)$$

and obviously follows from the gauge conditions for L^1, G^1 and L^2.

In the following table the multiplicity vectors for gauge invariant states of N-S representations are listed and are used extensively in calculations of degeneracies for colored particles.

n	1	8	10	$\bar{10}$	27	28	$\bar{28}$	35	$\bar{35}$	64	81	$\bar{81}$
0	1											
1/2		1										
1			1	1								
3/2					1							
2					2							
5/2								1	1			
3			1	1				1	1	1		
7/2								2	2	1		
4	1							1	1	1		
9/2										1	1	1

Table 8.4. Multiplicity vectors for N-S type color representations satisfying Super Virasoro conditions as function of conformal weight n. The table is needed in the calculation of masses of color excited leptons and colored bosons.

The construction proceeds in similar manner for Ramond type algebra and the following table lists the results.

n	1	8	10	10̄	27	28	28̄	35	35̄	64	80	80̄	81	81̄	125
0	1														
1															
2			1	1											
3					2										
4		2						1	1						
5			2	2						1					
6						1	1			2					
7													1	1	
8					1			4	4	1			1	1	1
9		68	46	46	68	4	4	30	30	28	1	1	5	5	2

Table 8.5: Table of multiplicity vectors for nonzero norm gauge invariant Ramond type color representations for various values of the conformal weight n. Table is needed in calculation mass of color excited states of quarks.

9 Appendix G: Information on $so(4)$ and $u(1)$ type Super Virasoro representations

The following tables give the degeneracies of operators creating nonzero norm states for $u(1)$ and $so..$ type representations.

(c,h)	Δ	0	1/2	1	3/2	2	5/2	3	7/2	4
(3/2,1/2)	N	1	1	1	2	3	4	5	7	10
(3/2,0)	N	1	0	0	1	1	1	1	2	2

Table 9.1. The number N of operators for $u(1)$ N-S type Super Virasoro representations for central charge $c = 3/2$ and vacuum weight $h = 1/2$ and $h = 0$ as function of the conformal weight Δ. In $h = 1/2$ case there are no singular vectors whereas in $h = 0$ case the operators $G^{1/2}$ and L^1 create zero norm states.

(c,h)	Δ	0	1/2	1	3/2	2	5/2	3	7/2	4
(0,1/2)	N	1	1	1	2	3	4	5	7	8

Table 9.2. The number N of operators for for $(c = 0, h = 1/2)$ N-S rep-

resentation as function of the conformal weight Δ. For $(c = 0, h = 0)$ representation all states created by Super Virasoro generators possess zero norm.

(c,h)	Δ	0	1	2	3	4
(0,1/2)	N	2	2	4	8	14

Table 9.3. The number N of operators for for $(c = 3/2, h = Q_K^2/2 = 1/2)$ Ramond representation as function of the conformal weight Δ. The representation appears in fermionic $u(1)$ sector.

(c,h)	Δ	0	1	2	3	4
(0,0)	N	1	2	4	8	14

Table 9.4. The number N of operators for $(c = 0, h = 0)$ $so(4)$ Ramond representation as function of the conformal weight Δ. The representation appears as basic building block of Kac Moody spinors. Note that G^0 creates zero norm state.

10 Appendix H: Number theoretic auxiliary results

The ground state degeneracies for fermions and bosons need not to be identical to their ideal values $D = 64$ and $D = 16$ and it is of interest to find under what conditions the degeneracy can be said to be near to its ideal value. This amounts to calculating the p-adic inverse of the D in general case. The calculation goes as follows.

a) The problem is to find the lowest order term in p-adic expansion of the inverse y of p-adic number $x \in 1, ...p - 1$. The remaining terms in expansion in powers of p can be found iteratively. The equation to be solved is

$$yx \quad = \quad 1 \ mod \ p \tag{37}$$

for a given value of x.

b) One can express p in the form

$$p = Nx + r \tag{38}$$

The evaluation of N and $r \in \{1, .., x - 1\}$ is a straightforward task. The defining equation for y can be written as

$$yx = m(Nx + r) + 1 = mNx + mr + 1 \tag{39}$$

From this one must have

$$mr + 1 = kx \tag{40}$$

and any pair (m, k) satisfying this condition gives solution to y:

$$y = mN + k \tag{41}$$

y must be chosen to be the smallest possible one.

Consider as examples two practical cases.

a) $p = M_n = 2^n - 1$ and $x = 15 = 2^4 - 1$. One obtains r by substituting repeatedly $2^4 = 1 \ mod \ x$ to the expression of M_n. M_n can be written in the form $M_n = 15(2^{n-4} + 2^{n-8} + ...) + r$ and the previous condition reads $mr + 1 = 15k$.

i) For M_{89} one has $r = 1$ and $(m, k) = (14, 1)$ giving $y = 14(2^{n-4} + 2^{n-8} + ...) + 1$. For the real counterpart of $Xp^2/2D$ one has the approximate expression $(7X \ mod \ 16)/15$ and approximately N-S mass formula for small quantum numbers results.

ii) For M_{127} and M_{107} one has $r = 7$ and $7m + 1 = 15k$ gives $(m, k) = 2, 1)$ and $y = 2(2^{n-4} + ...) + 1$. For $Xp^2/2D$ one has $X mod 16/15$: the factor 7 is absent.

b)$p = M^n$ and $x = 63 = 64 - 1$. One obtains r by substituting repeatedly $2^6 - 1 \ mod \ x$ to the expression of M_n. One has $r = 1$ for $n = 127$, $r = 31$ for $n = 107$ and $n = 89$. For the real counterpart R of Xp^2/D one has $R = (62X \ mod \ 64)/(63M_n)$ and $y = (60X \ mod \ 64)/(63M_n)$ for $n = 127$ and $107, 89$ respectively so that mass formulas change somewhat and in n-dependent manner if one has $D = 63$ instead of $D = 64$.

b) 1/5 factor appears in mass formulas for leptons and the previous argument leads to the expression $p^2/5 = (2^{126} - 2^{124} + 2^{122} - ..)p^2$. From this formula the real counterpart of, say 2/5, is in good approximation 4/5.